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VOLUME INTERSECTION OF TWO IDENTICAL
RIGHT CIRCULAR CONES

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The common volume of two cones representing the reverberation volume of a transducer pair included in a doppler flow meter was calculated. Variables in the calculation, written as a computer program, included the divergence angle of the cones, transducer separation, and transducer size. The volume was found to increase exponentially with an increase in the apex angle, ψ , of the cones for a fixed intersection angle, θ , limited to the condition $\psi < \theta$.		

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VOLUME INTERSECTION OF TWO IDENTICAL RIGHT CIRCULAR CONES

Ronald G. Hughes
J. Dean Clamons

INTRODUCTION

The successful operation of an instrument system depends on signal-to-noise levels as well as sensitivity. During the design phase of engineering hardware, a calculation is required to predict the applicability, within a particular set of physical conditions, of the total instrument package to the phenomena to be measured. This provides the constraints or limits, related to sensitivity and signal-to-noise level, which must be met by each of the instrument's system modules, and these serve as the goals for module parameters necessary to produce a functional instrument.

As applied to the doppler current meter, a calculation to determine the volume reverberation signal strength is made, based on the assumption used with sonar frequencies, that the reverberation signal strength is directly proportional to the common intersection volume of the beam patterns of a transmitter-receiver pair¹ (see Fig. 1). Concentric beams result in an infinite volume, excluding consideration of acoustic signal attenuation, but introduce the problem of testing the transducer system in a finite container since the reflection from the

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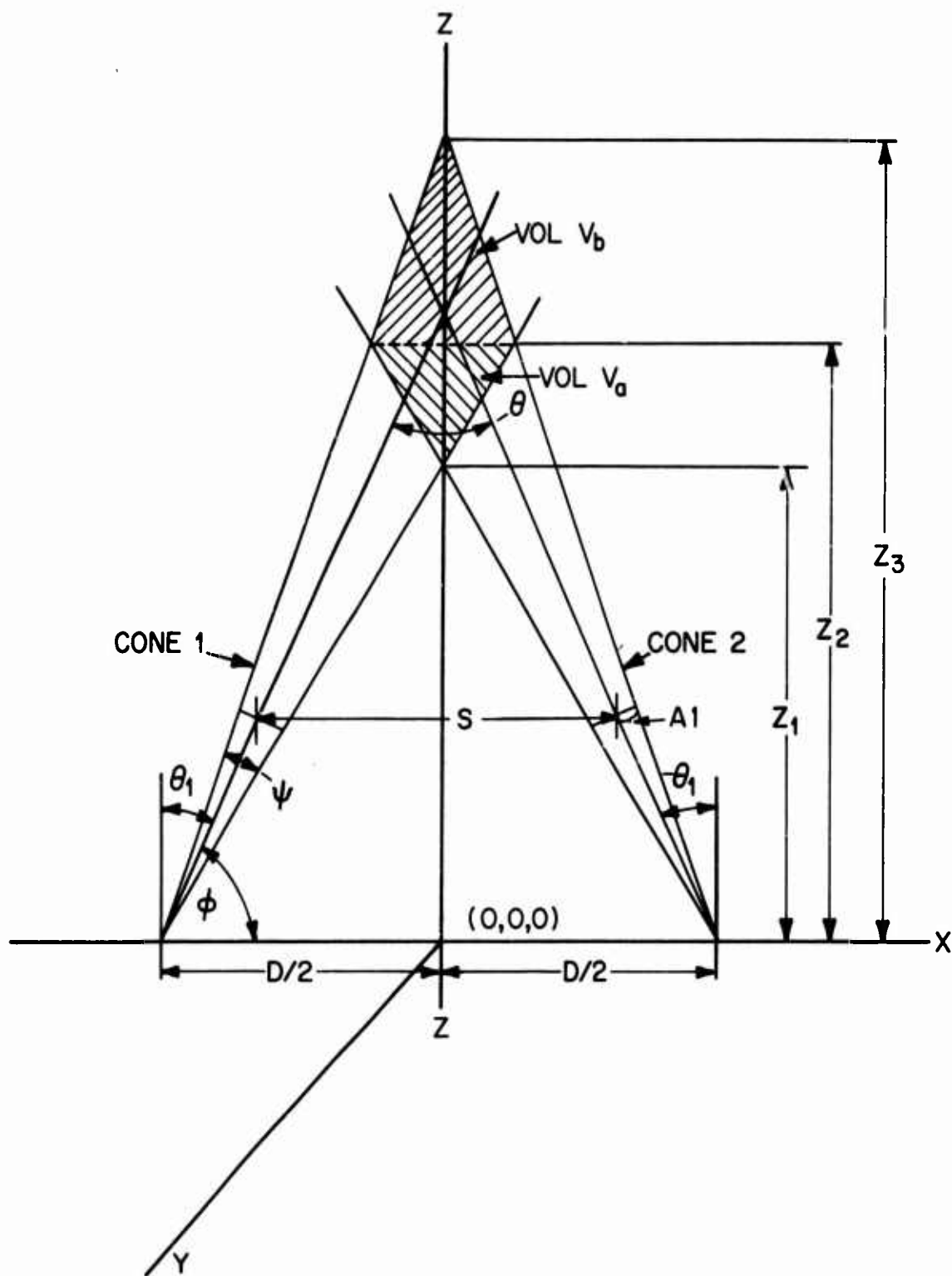


Fig. 1 - Two dimensional view of the volume intersection of two identical right circular cones of divergence angle ψ and intersection angle θ .

walls results in a noise signal which far exceeds the volume reverberation. But when the beams of the transducer pair are crossed, the desired signal is observed in the output of the signal processing electronics.

Following the above, the magnitude of the signal expected in the receiving transducer of the doppler meter configuration is obtained as a volume reverberation. This signal strength is assumed directly proportional to the common volume which is viewed by the receiving transducer and insonified by the transmitter, presupposing suppression of side lobes. The predicted value of the backscattered signal is calculated as a product of factors related to this common volume, the volume scattering function model, the beam pattern spreading, and the attenuation loss.

To implement this approach in predicting signal levels, the volume intersection of the doppler meter transducer pair is calculated with an assumption that both the radiated and received beam patterns are conical and identical.

MATHEMATICAL FORMULATION

The calculation of the volume intersection of two right circular cones is very simple in principle, but involves the tedious evaluation of a triple integral. Utilization of the symmetry of the problem in cylindrical coordinates fails since the area formed by the intersection of a cone with a plane not perpendicular to the symmetry axis is elliptical rather than circular.

Further restrictions imposed on the problem follow:

1. The vertices of the two cones are positioned on the x-y plane such that $Z = y = 0$.
2. The vertices are separated by a distance $D > 0$.

Variables include:

- θ , the angle of intersection of the symmetry axes of the two beams;
- ψ divergence angle of transducer beam (apex angle of the cone);
- A_1 , radius of the transducers; and
- S , separation of the centers of the transducers.

The analytic expression for a cone having its axis of symmetry congruent with the Z axis of an orthogonal Cartesian coordinate system is given by

$$x^2/a^2 + y^2/b^2 - \frac{z^2}{c^2} = 0 \quad (1)$$

For the right circular case eq. (1) changes from (see Fig. 2) to

$$x^2 + y^2 - C^2 z^2 = 0 \quad (2)$$

The constant C appearing in equation (2) is defined in terms of the divergence angle, ψ , specified for the transducers, as

$$C = \tan \psi / 2.$$

The expression for the rotated and translated cone in terms of the Cartesian coordinate system in Fig. 1 is determined by means of a rotation and translation operation applied to equation (2). The rotation operators about the y axis for angles $-\theta_1$ and $+\theta_1$ for cones 1 and 2 respectively are:

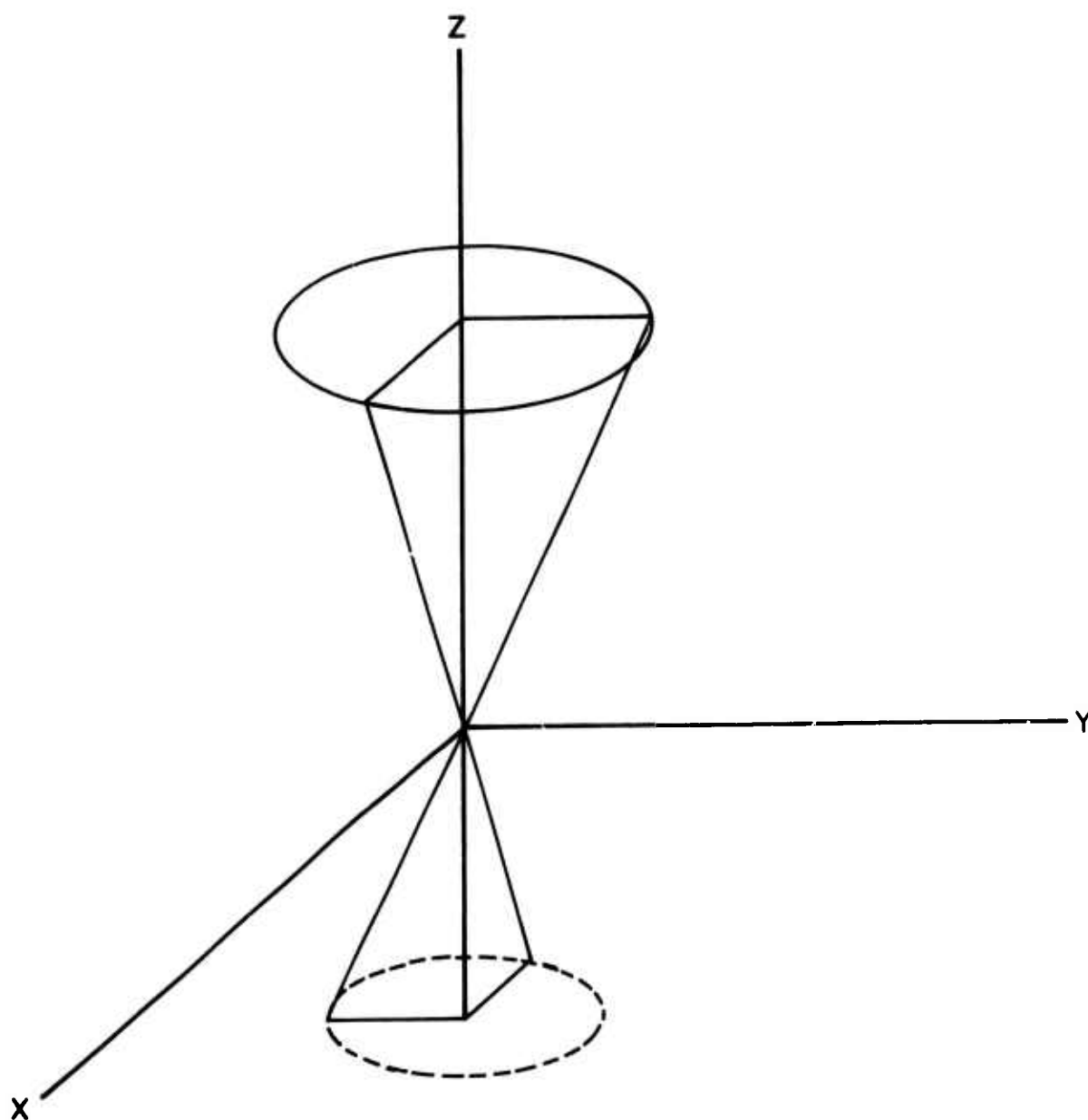


Fig. 2 - Three dimensional plot of the general expression for a right circular cone with vertex at the origin.

$$Ry_1 = \begin{pmatrix} \cos \theta_1 & 0 & -\sin \theta_1 \\ 0 & 1 & 0 \\ \sin \theta_1 & 0 & \cos \theta_1 \end{pmatrix}$$

$$Ry_2 = \begin{pmatrix} \cos \theta_1 & 0 & \sin \theta_1 \\ 0 & 1 & 0 \\ -\sin \theta_1 & 0 & \cos \theta_1 \end{pmatrix}$$

The angle θ_1 in the expressions for the rotation operators is defined in terms of angle ϕ as noted in Fig. 1: $\theta_1 = \phi - \pi/2$ for cone 1, and $\theta_1 = \pi/2 - \phi$ for cone 2. These substitutions in Ry_1 and Ry_2 result in:

$$Ry_1 = \begin{pmatrix} \sin \phi & 0 & -\cos \phi \\ 0 & 1 & 0 \\ \cos \phi & 0 & \sin \phi \end{pmatrix}$$

$$Ry_2 = \begin{pmatrix} \sin \phi & 0 & \cos \phi \\ 0 & 1 & 0 \\ -\cos \phi & 0 & \sin \phi \end{pmatrix}$$

Operating on equation (2) with Ry_1 and translating along the x axis by $-D/2$ yields for cone 1:

$$((x + D/2) \sin \phi - Z \cos \phi)^2 + y^2 = \tan^2 \psi/2 ((x + D/2) \cos \phi + Z \sin \phi)^2 \quad (3)$$

A similar application of Ry_2 and translation of $D/2$ on the axis produces the equation for cone 2 as

$$((x - D/2) \sin\phi + Z\cos\phi)^2 + y^2 = \tan^2 \psi/2 ((x - D/2) \cos\phi - Z\sin\phi)^2 \quad (4)$$

Because of the symmetry of the intersection volume with respect to the $x = 0$ and $y = 0$ planes, it is necessary to consider only that part of volume which lies in the first ($x - y$) quadrant. The volume under consideration can be further segmented into Volume V_a and V_b as shown in Fig. 1. Simplification of integration results in particular from setting the limits as V_a is bounded by cone 1 and the planes $x = 0$, $y = 0$, $Z = Z_2$ and V_b is bounded by cone 2 and the planes $x = 0$, $y = 0$, $Z = Z_2$. The integral in Cartesian coordinates for V_a is found to be:

$$\frac{V_a}{4} = \int_{Z_1}^{Z_2} dZ \int_0^{Z/\tan(\phi - \psi/2) - D/2} dx \int_0^{\left(\tan^2 \psi/2 ((D/2 + x) \cos\phi + Z\sin\phi)^2 - ((x + D/2) \sin\phi - Z\cos\phi)^2 \right)^{1/2}} dy$$

Integrating this expression with respect to the y variable and redefining the terms in the resulting integral as

$$A = \sin^2 \phi \tan^2 \psi/2 - \cos^2 \phi$$

$$B = 2 \sin\phi \cos\phi (\tan^2 \psi/2 + 1)$$

$$C = \cos^2 \phi \tan^2 \psi/2 - \sin^2 \phi$$

$$T1 = Z/\tan(\phi - \psi/2)$$

$$f(x) = \sqrt{x^2 C + Z B x - Z^2 A}$$

results in the following expression:

$$\frac{V_a}{4} = \frac{1}{\sqrt{-C}} \int_{Z_1}^{Z_2} \frac{dZ}{2} \sqrt{\frac{Z^2(B^2 - 2AC)}{-4C} - x^2 - \left(\frac{Z^2(B^2 - 4AC)}{-8C}\right)} \sin^{-1} \left(\frac{2x\sqrt{-C}}{Z\sqrt{B^2 - 4AC}} \right) \Bigg|_{f(D/2)}^{f(T1)}$$

The same equation yields $V_b/4$ with the substitutions

B -B

D -D

T1 -Z/tan($\phi + \psi/2$)

and the total volume of intersection is obviously $V = 4(V_a/4 + V_b/4)$.

RESULTS

The intersection volume was calculated for a fixed transducer spacing S and radius A_1 , for varying beam divergence angle ψ , and beam intersection angle θ and the results are plotted in Fig. 3. Inspection of the plot indicates that the intersection volume differs from an exponential function as the beam intersection angle approaches the beam divergence angle. This result is particularly noticeable for the parameter $\theta = 6.5^\circ$. The plots for the two other beam intersection angles are non-exponential to a lesser extent over the range of values taken for ψ .

In order to check the numerical integration method, a Monte Carlo program was written to calculate the intersection volume for $\theta = 8^\circ$ and a range of ψ from 2° to 4.5° . For this calculation, the transducer

separation was fixed at 3.81 cm and the transducer radii at 0.65 cm. The nearly exponential dependence of the calculated volume as a function of ψ is plotted in Fig. 4. The plot shows the comparison of the numerical integration and Monte Carlo methods to be nearly equivalent given the same beam geometry. The calculated intersection volumes found by these two methods indicate that the results of an earlier study² are too liberal in approximating the size of the assumed elliptical cross section.

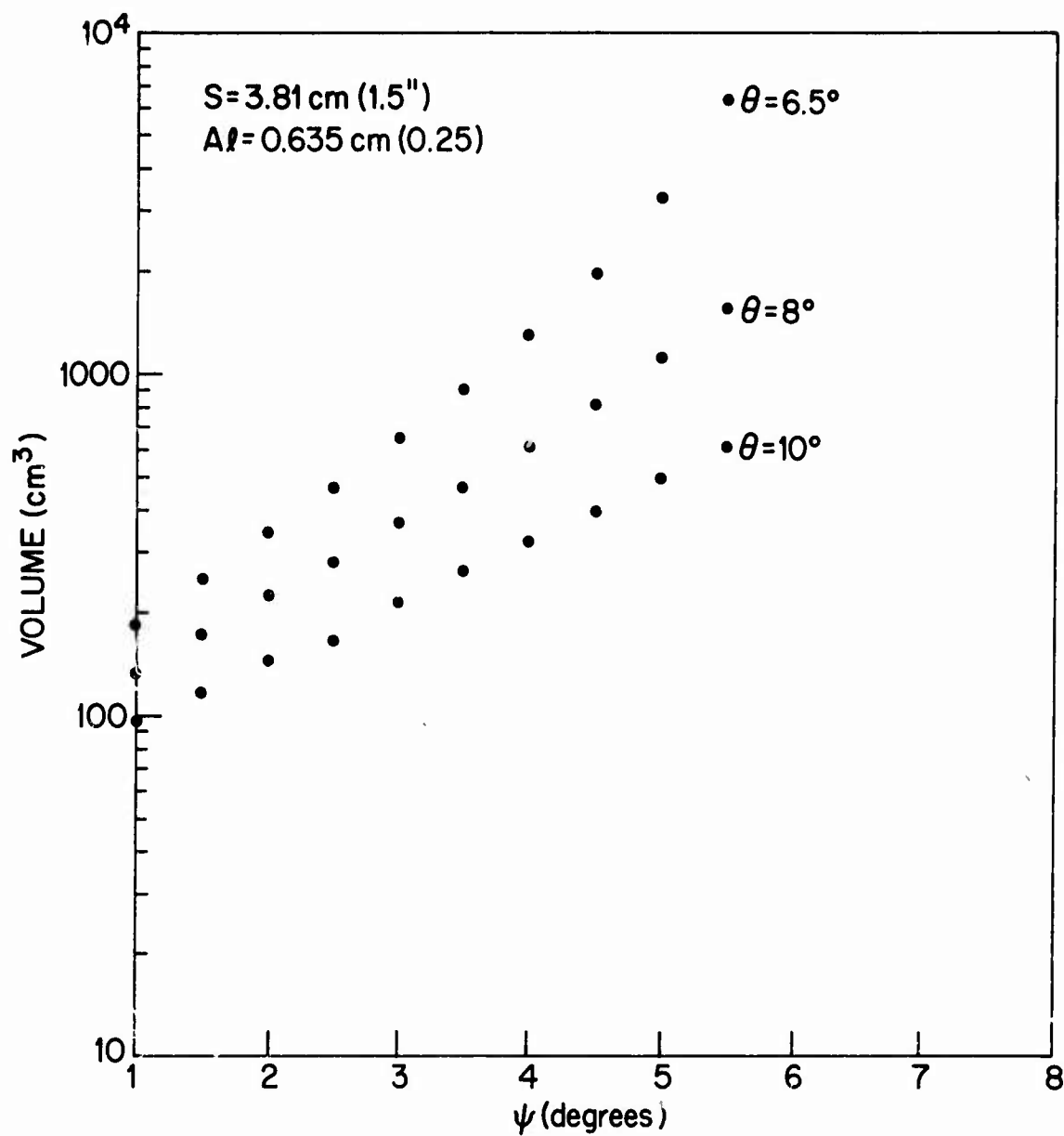


Fig. 3 - Intersection volume versus divergence angle ψ for transducers of .635 cm radius and 3.81 cm separation center to center.

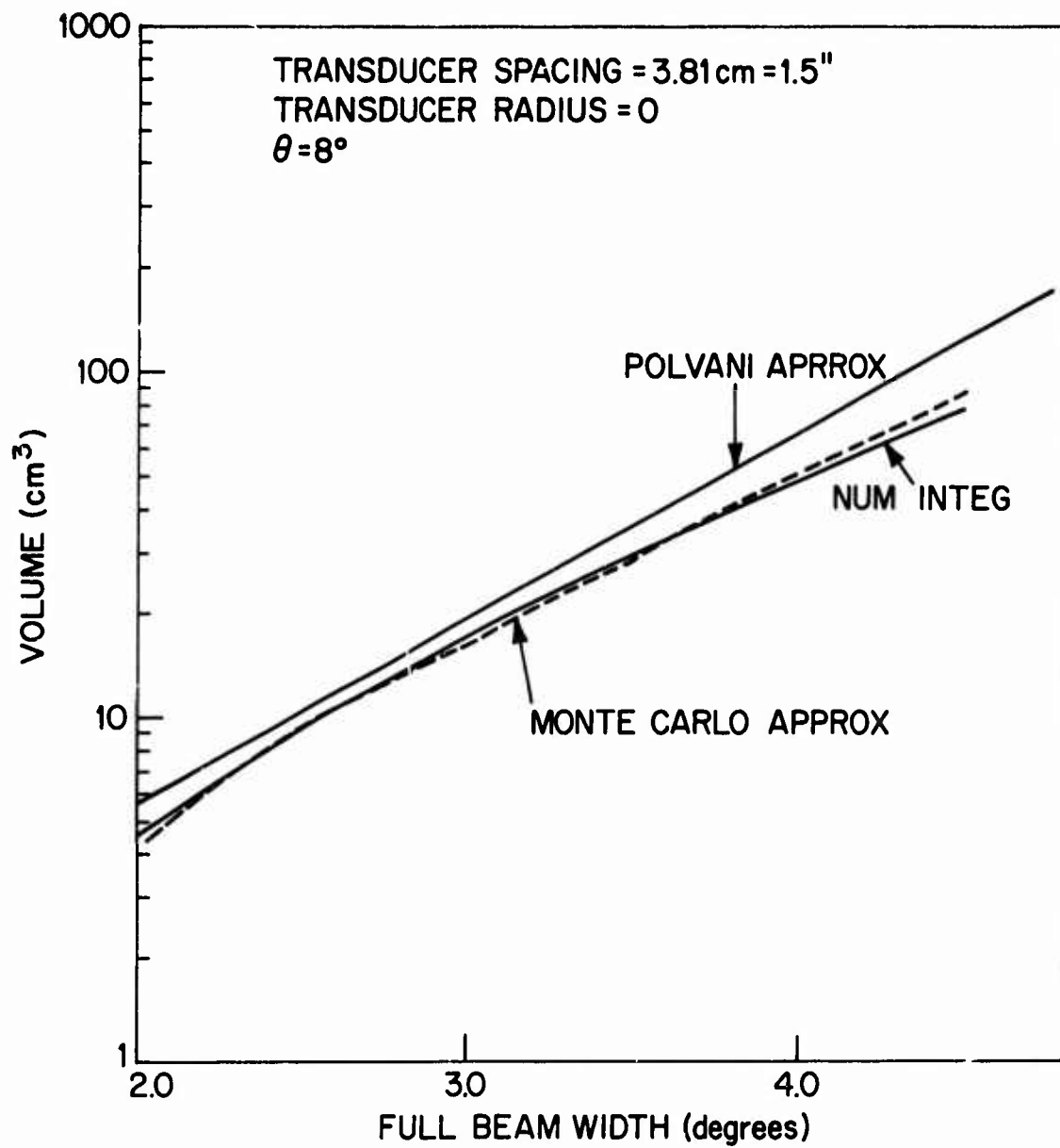


Fig. 4 - Comparison of the calculated intersection volumes as a function of full beam width.

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1. Physics of Sound in the Sea (Part II Reverberation) Research Analysis Group (Committee on Undersea Warfare) National Research Council.
2. D.G. Polvani, Ocean Engineering Memo #73-04 (11 Jan 1973) Westinghouse Electric Corporation, Annapolis, Md.

APPENDIX I

This appendix includes a copy of the FORTRAN IV coded program which was used to perform the intersection volume calculations. Instructions for operating the program are included in comment statements listed on the printed output. In addition, definitions of the input constants of transducer size, intersection angle, and divergence angle of the acoustic beam are listed.

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PROGRAM VOLUME

00100 PROGRAM VOLUME(INPUT,OUTPUT)

00110* THIS PROGRAM CALCULATES THE VOLUME OF INTERSECTION OF
00120* THE BEAM PATTERNS OF TWO IDENTICAL TRANSDUCERS. THE BEAM
00130* PATTERNS ARE ASSUMED TO BE CONICAL WITH AN INCLUDED ANGLE
00140* OF PSI DEGREES, AND THEIR AXES INTERSECT AT AN ANGLE OF
00150* THETA DEGREES. THE RADIUS OF THE TRANSDUCERS IS A1 AND
00160* THE SEPARATION BETWEEN TRANSDUCER CENTERS IS S.

00170* THE VOLUME IS FOUND BY CALCULATING THE AREA OF THE
00180* HORIZONTAL CROSS-SECTION AND NUMERICALLY INTEGRATING THE
00190* AREA WITH RESPECT TO THE Z-COORDINATE USING A ROMBERG
00200* INTEGRATION SUBROUTINE TO GET THE VOLUME.

00210* FOR DERIVATION OF THE FORMULAS AND A MORE DETAILED
00220* DESCRIPTION OF THE GEOMETRY SEE:

00230*

00240* INPUTS: PSI IN DEGREES
00250* THETA IN DEGREES
00260* A1 IN ANY UNITS
00270* S IN THE SAME UNITS AS A1
00280* OUTPUTS: VOLUME IN THE SAME UNITS AS A1

00290*

00300*

00320 COMMON TEMP,C,B,A,D,T2

00330 EXTERNAL FCN1

00340 30 PRINT,*INCLUDED ANGLE OF TRANSDUCER BEAM PATTERN (PSI)*

00350 READ,PSI1

00360 PRINT,*ANGLE OF INTERSECTION OF BEAMS (THETA)*

00370 READ,THETA1

00380 PRINT,*RADIUS OF TRANSDUCERS (A1)*

00390 READ,A1

00400 PRINT,*DISTANCE BETWEEN TRANSDUCER CENTERS (S)*

00410 READ,S

00420* CALCULATE PHI (THE ANGLE OF TILT OF THE CONES) AND CONVERT

00430* ALL ANGLES TO RADIAN. FOR CALCULATION PURPOSES PSI/2

00440* IS USED.

00450 THETA=THETA1*3.14159/180.0

00460 PHI1=(180.0-THETA1)/2.0

00470 PHI=PHI1*3.14159/180.0

00480 PSI=PSI1*3.14159/180.0

00490 PSI=PSI/2.0

00500* CALCULATE D (THE DISTANCE BETWEEN VERTICES OF THE CONES)

00510* CALCULATE Z (DISTANCE FROM PLANE OF VERTICES TO INTERSECTION

00520* OF AXES.

00530 X=A1*SIN(THETA)/TAN(PSI)

00540 D=2.0*X+S

00550 Z=D*TAN(PHI)/2.0

00560* IF PSI+PHI>90 DEGREES VOLUME IS INFINITE

00570 IF(PSI+PHI.GT.3.14159/2.0)GO TO 1

00580* CALCULATE LIMITS OF INTEGRATION (Z1,Z2,Z3) AND CONSTANTS

00590* USED IN CALCULATION (A,B,C,TEMP,T2)

00600 S1=SIN(PHI)

00610 C1=COS(PHI)

00620 T=TAN(PSI)

00630 T2=TAN(PHI-PSI)

00640 A=S1*T*S1*T-C1*C1

00650 B=2.0*S1*C1*(T*T+1.0)

00660 C=C1*T+C1*T-S1*S1

00670 TEMP=4.0*T*T

00680 Z1=D*T2/2.0

```

00690 Z2=D*SIN(PHI+PSI)*SIN(PHI-PSI)/SIN(2.0*PHI)
00700 Z3=D*TAN(PHI+PSI)/2.0
00710*   CALCULATE VA (LOWER VOLUME)
00720 CALL XCROME(Z1,Z2,10E-8,VA,IERR,FCN1)
00730*   CHANGE VARIABLES FOR CALCULATING VB
00740 D=-D
00750 F=-F
00760 T2=-TAN(PHI+PSI)
00770*   CALCULATE VB (UPPER VOLUME)
00780 CALL XCROME(Z2,Z3,10E-8,VB,IERR,FCN1)
00790*   CALCULATE TOTAL VOLUME AND PRINT RESULTS
00800 V=4.0*(VA+VB)
00810 F=-D
00820 PRINT 500,PSI1,THETA1,PHI1,S,A1,D,V
00830 500 FORMAT(*PSI= *,F10.2,*   THETA= *,F10.2,*   PHI= *,F10.2,
00840+   /S= *,F10.2,*   A1= *,F10.2,*   D= *,F10.2/
00850+   *VOLUME OF INTERSECTION = *,E15.8)
00860 GO TO 20
00870 1 PRINT 501,PSI1,THETA1,PHI1,S,A1,D
00880 501 FORMAT(*PSI= *,F10.2,*   THETA= *,F10.2,=   PHI= *,F10.2/
00890+   *S= *,F10.2,*   A1= *,F10.2,*   D= *,F10.2/
00900+   *VOLUME OF INTERSECTION IS INFINITE*)
00910 20 PRINT 501
00920 505 FORMAT(//)
00930 GO TO 30
00940 END
00950 FUNCTION FCN1(Z)
00960*   THIS FUNCTION CALCULATES THE AREA OF CROSS-SECTION AT
00970*   HEIGHT Z. IT FIRST CALCULATES THE LIMITS AT WHICH THE
00980*   AREA INTEGRAL MUST BE EVALUATED AND THEN CALLS FCN2 AT
00990*   THESE LIMITS TO MAKE THE EVALUATIONS.
01000 COMMON TEMP,C,E,A,D,T2
01010 Y1=D/2.0
01020 Y2=Z/T2
01030 Y1=C*Y1*Y1+Z*Y1*B+Z*Z*A
01040 Y2=C*Y2*Y2+Z*Y2*B+Z*Z*A
01050 IF(ABS(Y1).LT.10E-10)Y1=0.0
01060 IF(ABS(Y2).LT.10E-10)Y2=0.0
01070 IF(Y1.LT.0.0)PRINT,*,Y1:2=*,Y1
01080 IF(Y2.LT.0.0)PRINT,*,Y2:2=*,Y2
01090 Y1=SQRT(ABS(Y1))
01100 Y2=SQRT(ABS(Y2))
01110 FCN1=FCN2(Y2,Z)-FCN2(Y1,Z)
01120 RETURN
01130 END
01140 FUNCTION FCN2(X,Z)
01150*   THIS FUNCTION EVALUATES THE AREA INTEGRAL AT X,Z.
01160 COMMON TEMP,C,E,A,D,T2
01170 RAD=Z*4*TEMP/(-4.0*C)
01180 F=X/SQRT(RAD)
01190 F2=RAD-X*X
01200 IF(F.LE.1.0) GO TO 1
01210 PRINT,*,ARG OF ASIN > 1*,F
01220 F=1.0
01230 F2=0.0
01240 1 FCN2=(A*SQRT(F2)-RAD*ASIN(F))/(SQRT(-C)*2)
01250 RETURN
01260 END
READY.

```